TABLE I
COMPARISON OF CW AND PULSE DUPLEXER CHARACTERISTICS

Duplexer	Ferrite/Diode		Balanced		Modified Branch	
	cw	Pulse	cw	Pulse	cw	Pulse
Transmit Loss	.4 db	.8 db	1.5 db	1.3 db	1.2 db	5.3 db
Rec. Loss	1.6 db	2.1 db[a]	2.1 db	2.7 db[a]	1.7 db	2.5 db[a]
Power Handling	—	> 20 kw	—	18 kw	—	> 20 kw
Rec. Isolation	>57 db	> 57 db	>50 db	>50 db	>50 db	> 50 db
Switching (includes driver)	135 ns	135 ns	135 ns	135 ns	135 ns	135 ns
Pulse Fidelity	—	good	—	fair	—	poor

[a] For low-level input.

suited to short-pulse operation due to the high transmit loss of 5.3 dB. This is caused by the ineffectiveness of the diode-loaded stubs that are required to introduce a shunt open circuit across the transmitter-to-antenna branch of the structure. The electric field build-up time and the nonuniformity of the pulse amplitude are not conducive to establishing the appropriate field conditions.

A brief summary and comparison of the short-pulse and CW performance of the duplexers are given in Table I. Of the several duplexers investigated, the combination of ferrite circulator and shunt-mounted diode switch is most suited to duplexing of short-pulse signals. It offers high-fidelity transmission with the least attenuation of transmit and receive signals.

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Addenda to "The Exact Dimensions of a Family of Rectangular Coaxial Lines with Given Impedance"

HENRY J. RIBLET

When the above paper[1] was written, it was the writer's opinion that the Bowman [1] reference was the best available. Quite by chance, while searching the literature in another connection, he encountered a reference made by Anderson [2] to a paper by Bergmann [3] in which the same problem is solved with the same results and methods. Oddly enough Bergmann's paper, considering the general case, predates Bowman's solution of the special case by ten years.

There are a few minor differences which may be noted at this time as follows.

1) Bergmann's restriction [3, eq. (1)[1]] that $\lambda < k < 1$, is more restrictive than my comparable condition $0 < \alpha < \beta$ and $\alpha\beta < 1$, since the condition $\beta < 1$ ignores one of two possible orientations of the figure.

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The author is with Microwave Development Laboratories. Inc., Needham, Mass.
[1] H. J. Riblet, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 538–541, Aug. 1972.

2) My evaluation of the multivalued elliptic integrals by plotting the path of integrations on the $R(k)$ and $R(\lambda)$ surfaces appears to represent a substantial simplification of the solution.

3) Bergmann has given necessary and sufficient conditions that a given system of concentric rectangles belong to the family under consideration. Applying arguments similar to Bergmann's to equation (11),[1] we have

$$\frac{\overline{OA} + \overline{CB}}{\overline{AB}} = \frac{K(\lambda)}{K'(\lambda)} \quad \text{and} \quad \frac{\overline{DC} + \overline{EO}}{\overline{DE}} = \frac{K'(k)}{K(k)}. \tag{A}$$

Thus the ratios of the dimensions of any rectangular system uniquely determine λ and k. In addition,

$$\frac{\overline{AB}}{\overline{DE}} = \frac{K'(\lambda)}{K(k)}. \tag{B}$$

Now, if this condition is satisfied after λ and k have been determined from the dimensions of the given system of concentric rectangles, it is not difficult to show that the α and β determined by (3)[1] have the property that $0 < \alpha < \beta$ and $\alpha\beta < 1$, and moreover that the rectangles given by equation (11)[1] differ at most by a scale factor from the given rectangles. Thus condition (B) is sufficient as well as necessary.

Perhaps it is even more useful to observe that the solution given first by Bergmann and then the writer can be extended to map the upper half of the complex plane into the interior of a symmetrical U-shaped region. This permits the determination of the characteristic impedance of the same class of concentric rectangular conductors with the difference that one or more walls may be magnetic in an asymmetrical manner.

To see how this is possible, consider the transformation,

$$z = \int_0^u \frac{M(1-u^2)^{1/2}\,du}{[(a^2-u^2)(b^2-u^2)(c^2-u^2)]^{1/2}} \tag{1}$$

which maps the upper half of the u plane in Fig. 1 into the interior of the U-shaped figure in the z plane which is symmetrical about the imaginary axis, for M real and positive. Then the transformation $v = u^2$ maps the upper left-hand quadrant of the u plane into the lower half of the v plane. Consequently, when this substitution is made in (1), it follows that

$$z = \frac{M}{2}\int_0^v \frac{(1-v)^{1/2}\,dv}{[v(a^2-v)(b^2-v)(c^2-v)]^{1/2}} \tag{2}$$

maps the lower half of the v plane into the L-shaped figure in the z plane which is just that half of the U-shaped figure which lies in the left-half plane, since values on the negative v axis correspond to imaginary values of z. Here again, corresponding boundary points are denoted by the same capital letters.

Now the further transformation

$$v = \frac{1-w}{1-\alpha\beta w} \tag{3}$$

maps the lower half of the v plane onto the upper half of the w plane whenever $\alpha\beta < 1$. Then, if we select a, b, and c so that

$$a^2 = \frac{1+\alpha}{\alpha(1+\beta)} \quad b^2 = \frac{1+\beta}{\beta(1+\alpha)} \quad c^2 = \frac{1}{\alpha\beta} \tag{4}$$

the substitution of (3) in (2) results in the transformation

$$z = M'\int_1^w \frac{w^{1/}\;dw}{[(1-w)(1+\alpha w)(1+\beta w)(1-\alpha\beta w)]^{1/2}} \tag{5}$$

which maps the upper half of the w plane into the same L-shaped figure in the z plane. However, the coordinates of the points on the real axis of the w plane corresponding to the corners of the figure in the left half of the z plane are given in terms of α and β. Here $M' = M\alpha\beta(1+\alpha)^{1/2}(1+\beta)^{1/2}/2$ and any ambiguity of sign is resolved by noting that the integral in (5) differs at most by an additive constant from an integral, equation (1)[1], which is known to map the upper half w plane into an L-shaped region in the z plane having this orientation.

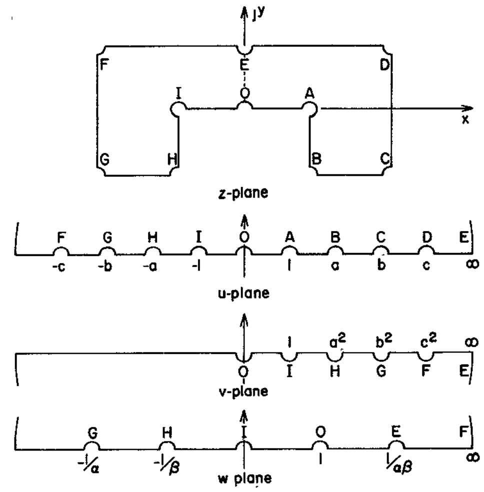

Fig. 1.

It is clear then that the dimensions of the U-shaped rectangular section of Fig. 1 are given in terms of α and β exactly as in footnote 1. We are now, however, in a position to determine the capacitances of certain asymmetrical arrangements where the U-shaped figure is bounded by electric and magnetic walls. Two of the many possible cases are considered as follows.

1) All the boundary walls are electric except for CD which is magnetic.

We are then concerned, in the u plane, with the capacitance of the line segment HB with respect to the line segment GD. As is well known, this capacitance is given by $K'(k_0)/K(k_0)$ where

$$k_0^2 = \frac{(c-a)(b-a)}{(c+a)(b+a)}$$

$$= \frac{[\beta^{1/2}(1+\alpha) - \alpha^{1/2}(1+\beta)][(1+\alpha)^{1/2} - \alpha^{1/2}(1+\beta)^{1/2}]}{[\beta^{1/2}(1+\alpha) + \alpha^{1/2}(1+\beta)][(1+\alpha)^{1/2} + \alpha^{1/2}(1+\beta)^{1/2}]}. \quad (6)$$

2) All the boundary walls are electric except for CD and FD which are magnetic.

We are then concerned, in the u plane, with the capacitance of the line segment HB with respect to the line segment FG. This capacitance is given by $K(k_1)/K'(k_1)$ where

$$k_1^2 = \frac{2a(c-b)}{(a+b)(c-a)}$$

$$= \frac{[(1+\beta)^{1/2} - \beta^{1/2}(1+\alpha)^{1/2}]2\alpha^{1/2}(1+\beta)}{[(1+\alpha)^{1/2} - \alpha^{1/2}(1+\beta)^{1/2}][\alpha^{1/2}(1+\beta) + \beta^{1/2}(1+\alpha)]}. \quad (7)$$

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Correction to "Fast Parameters Calculation of the Dielectric-Supported Air-Strip Transmission Line"

EUGENIO COSTAMAGNA

In the above letter,[1] on page 156, second column, line 1, the words "relative phase velocity" should read "phase velocity in m/s $\times 10^{-9}$."

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The author is with Marconi Italiana S.p.A., Genoa Cornigliano, Genoa, Italy.

[1] E. Costamagna, *IEEE Trans. Microwave Theory Tech.* (Lett.), vol. MTT-21, pp. 155–156, Mar. 1973